Homework for Week 8
Nominal due date: Friday, 16 October


11–15. 10.1Q (by “impulse,” the book means change in momentum), 10.1E, 10.15P, 10.37P, 10.59P

Answer to week-7 question not from the book

6. This problem was too hard and too involved. If you got it, you'll do very well on the quiz. The compression of the spring, x, (i.e., the amount by which it is displaced down from height b) due to the weight of mass a is given by setting potential energy stored in the compressed spring equal to the difference in gravitational potential energies through which the cough drop falls, since the mass starts and finishes without kinetic energy: thus \( \frac{1}{2} ax^2 = agx \), where g is the gravitational acceleration. Since \( x \neq 0 \), I assume \( d > b - x \) and that the mass slides along the board rather than rolling. To move a at all, the spring must overcome the combined forces of gravity and static friction. Let the x axis lie along the board, the y axis normal to the board. Then the following forces act: spring = \( \pm cx \); gravity = \( -xag \cos(f) - yag \sin(f) \); normal = \( N \); maximum static friction = \( -x\mu N \). Since normal acceleration is zero, \( N = ag \sin(f) \). Thus to get the mass moving requires \( cx > ag \cos(f) + \mu N \), giving (after substitution for x and N) \( \mu \leq \frac{2ag \cos(f)}{ag \sin(f)} \). Assume this holds.

Energy is dissipated in sliding friction while the spring drags the mass along the board and continues to be dissipated (if \( d > b \)) while the mass rises up the board after losing contact with the spring. In either case, the mass must slide a distance \( d - b + x \) along the board. Setting the energy dissipated in friction plus the energy stored as gravitational potential while the mass rises equal to the energy stored in the spring, we have \( \mu K N (d - b + x) + ag (d - b + x) \cos(f) = \frac{1}{2} cx^2 \). Solving for the coefficient of kinetic (sliding) friction and plugging in the previously determined values for x and N, we find \( \mu K = \frac{3ag}{mg \cos(f) + ag \sin(f)} - \cot(f) \). (One easily verifies that this has no units, which is correct for coefficients of friction. The divergence as \( f \to 0 \) simply indicates that friction is always ineffective when no normal force acts.) In the case that \( d < b \), we require that the maximum static friction which takes over as soon as the drop stops moving should exceed the gravitational force less the spring force: \( \mu N \geq ag \cos(f) - c(b - d) \), or \( \mu \geq \frac{ag \cos(f) - c(b - d)}{ag \sin(f)} \). Finally, in the case that \( d > b \), there is no spring force acting on the cough drop when it reaches the top of the ironing board, so static friction must simply counteract gravity: \( N \mu K \geq ag \cos(f) \), or \( \mu \geq \cot(f) \).

Some things to mull over in advance of the test: what is the result of hooking springs to springs? Where can momentum hide? Where can energy hide? What is the relation of acceleration to speed for an object going around in a circle? What force is responsible for providing this acceleration? If a potential energy depends only on position, what is the force? How did we motivate conservation of energy and momentum?

Some general reminders: Don’t memorize signs; you can usually figure out which way something is supposed to go. Don’t memorize whether a formula has a sine or a cosine in it; you can always figure out which it has to be by considering angles such as 0° and 90°. Fundamental formulas, such as Hooke’s law and the definitions of momentum and kinetic energy, will be on the front cover of the quiz.