Computational-Physics

Homework for Week 6

Due on that midnight that falls between Tuesday, 5 October and Wednesday, 6 October, 2004

The goal this week is to answer problem 1 on page 98 of Giordano. These suggested steps outline the solution.

1. Write a program to integrate the motion of a planet. At aphelion or perihelion of an ellipse, velocity is perpendicular to radius from the orbital center (Sun, approximately), so it is enough to input initial position $x$ and initial velocity $v^y$, with $y = v^x = 0$. Other inputs are timestep (or number of evaluations), total time of simulation, and the non-Newtonian constants $\beta$ (4.8) and $\alpha$ (4.9). Initially, outputs should be time, $x$ position, and $y$ position. You might as well also output $r = \sqrt{x^2 + y^2}$, since the integrator has to calculate this anyway. Using the initial conditions worked out by Giordano on page 95, generate a picture of Mercury’s (Newtonian) orbit over a few periods without visible precession.

2. It is easier to track aphelion than perihelion. Using the output arrays from part 1, determine when $|r|$ stops getting bigger and starts getting smaller. You should use linear interpolation in order to improve the precision of your estimate. Sticking for now with Newtonian gravity (and no other planets), we’ll still find that the orbit precesses if the timestep is too large. Plot the rate of unphysical precession against timestep in order to settle on a good timestep for the rest of the simulation.

3. Verify that a small change in the inverse-square law away from $\beta = 2$ (try, for instance, $\beta = 2.1$) leads to precession. Do the same for $\alpha > 0$.

4. See if you can reproduce Giordano’s figure 4.9 for the unphysically-large value of $\alpha$.

5. The slope of figure 4.9 gives the precession rate (in, e.g., seconds of arc per [Earth] century) for that value of $\alpha$. To estimate the slope, you will need a least-squares fitting subroutine. Although Matlab has one built in (see Pratap), I want you to write your own instead. See Appendix 3 of Giordano or section 15.2 of Numerical Recipes. We’ll work on this in class on Wednesday. Demonstrate your subroutine with some small sets of made-up data consisting of $x$ and $y$ values having nothing to do with the astrophysical problem.

6. Apply part 5 to the graph in part 4 to estimate the precession rate for that $\alpha$.

7. Now, generate plots similar to figure 4.9 for a few other values of $\alpha$. Repeat part 6 for each. Plot the precession rate as a function of $\alpha$. Fit this to a straight line. This gives you an estimate for $C$ on page 94. (If you wish to be fancy, you can modify the least-squares algorithm to force the line to pass through the origin, but this is not required.)

8. From part 7, determine the precession of the aphelion (thus also of the perihelion) of Mercury due to weak-field general-relativistic effects in arcseconds per [Earth] century. Use $\alpha = 1.1 \times 10^{-8}$ AU$^2$. 