Problem worked by Datta to prepare (don’t turn in): 4.1
To turn in:
1. (R.E. Prange) Show that for the conditions of a quantum Hall plateau, the conductivity (not conductance) tensor may be written

\[
\sigma = \begin{pmatrix}
0 & -je^2/h \\
je^2/h & 0
\end{pmatrix},
\]

where \( j \) is a small integer and \(-|e|\) is the electron charge (Gaussian units). Now show that it follows that the measured Hall and longitudinal resistances (not just resistivities) are independent of sample geometry (i.e., the distances between probes).

2. (R.E. Prange) Datta’s figure 1.6.4 shows basis states that are confined in the \( y \) direction but extended in the \( x \) direction; however, because of the large degeneracy of Landau levels, eigenstates that are confined in both directions can be formed as linear combinations of the basis states. Show that in the gauge \( A = -(1/2)r \times B \) the eigenstates of the lowest Landau level can be written

\[
\psi(x, y) \propto z^m \exp(-|z|^2/4),
\]

where \( z = x - iy \) and \( m \) is the negative angular momentum about the \( B \) field (perpendicular to the sample).