Read the syllabus. You are allowed to consult textbooks, but you are not allowed to copy directly from them or from others; neither may you consult solutions from a previous year, nor any “solutions” manual. When turning in your homework, list any sources you consulted; you will not lose any credit for doing so. For example, you could write something like “The following solution uses an idea for deriving the Maxwell relations from Reichl, chapter 1.” Textbook abbreviation: K&K=Kittel and Kroemer.

Reading: K&K chapter 1, start chapter 2. Section 2.2 in Mandl is relevant for this week. Note the misprint in the binomial-coefficient symbol in Mandl equation (2.2): there should be no horizontal line.

1. Chapter 1 of K&K derives the multiplicity $g(N, s)$ for the binary spin model. The obvious generalization to moments that can take more than two values is surprisingly difficult, so I’ll ask you to evaluate the multiplicity only for specific, small numbers. Consider $N$ moments, each of which can take the value $+1, 0, \text{ or } -1$. (Such a system comes up in quantum mechanics when the moments are spin-1 particles as opposed to spin-1/2 particles, which give the binary system considered in chapter 1.) The maximum total moment is $N$. For example, for $N = 2$ spins, there are three ways to get total moment zero: $0_1 0_2$, $(+1)_1 (-1)_2$, and $(-1)_1 (+1)_2$. How many ways are there to get total moment zero for $N = 4$ and $N = 6$? How many ways are there to get a total moment $N/2$ for $N = 4$ and $N = 6$? Is this numerical evidence (admittedly scant) consistent with the idea that, in the limit of large $N$, fluctuations away from the most probable configuration become rare? Extra credit for a general formula for $g(N, s)$ in closed form that does not involve a hypergeometric function, but I don’t know whether it’s possible.

2. A coin is tossed 800 times. (a) Using the Gaussian approximation of chapter 1 in K&K, calculate the probability of getting heads exactly 380 times. You could write down the exact binomial coefficient, but you would have to be exceedingly careful in order to avoid numerical overflow. The answer, to compare to your Gaussian approximation, is about 1.03831%. (b) What is the probability of getting heads 380 times or fewer (i.e., no times or 1 time or 2 times or ... 380 times)? Again, use the Gaussian approximation. You will need to evaluate the error function $\text{erf}(x) = \left(2/\sqrt{\pi}\right) \int_0^x \exp(-t^2) dt$, for which you can use a computer program like Mathematica or Maple, or you can look it up in a book of tables, such as that by Abramowitz and Stegun. If you have trouble, let me know.

3. K&K 2.4 (it’s in chapter 2 but also makes sense in chapter 1).