Solid State I

Homework (#6) for Weeks 7 and 8

Due Friday, 10 March, 2006

I’ll talk about the practice midterm the week before the test, 6 March.


1b. Consider an electron initially at the bottom of the band \((k = 0)\) accelerated by a constant electric field. A free electron would follow an unbounded trajectory in real space, but the Bloch electron instead oscillates: thus a D.C. voltage gives rise to an alternating current.\(^1\) For a D.C. field in the \((1 0)\) direction, calculate and sketch the electron’s trajectory as a function of time. (Unfortunately, the Ziman program does not do this for you.)

1c. The trajectory of part 1b is closed, that is, repeats itself (periodically). By contrast, the orbit in Preset 3, while bounded, is not closed. What is the condition for a closed orbit in real space?

2a. A&M 12.2(a). Without loss of generality, call the direction of the magnetic field the \(\hat{z}\) axis. If you’re having trouble doing this problem generally, you may with first to attempt the following special case, which is very much easier: assume the magnetic field is pointing along one of the principal axes of the inverse-mass tensor \(M^{-1}\), so that you can take the inverse-mass tensor to be diagonal, \(M^{-1}_{xx} = M^{-1}_{yy} = M^{-1}_{zz} = etc. = 0\). For full credit, do not make this assumption. You may, however, rotate about the \(\hat{z}\) axis so that \(M^{-1}_{xy} = M^{-1}_{yx} = 0\).

2b. A&M 12.2(b)

2c. S&D 9.26

3. A&M 12.7 (one-word answers, but you need to justify them).

4a. After skimming through the exercises in S&D pages 198–205, see if you can reproduce the \((-v_e)\) (squares) trace from figure 10.1. (S&D left off the minus sign in the figure inset.) The horizontal axis is controlled by the Fermi energy; you need not sample quite so many points as S&D did, and you may extract the average velocities by eye, if you prefer. (It might still be helpful to show some sort of error bars.) Be sure the hole display is switched off, or else the graph will show hole velocities instead of electron velocities.

4b. From the curve in 4a, deduce the current trace (diamonds in figure 10.1).

4c. Switch on the hole display; the origin doesn’t matter (except that for some reason the green ball appears only with the shifted origin). See if you can reproduce the trace of average hole velocity (triangles in figure 10.1).

4d. From the curve in 4c, deduce the current trace.

4e. Which of the following is the correct way to measure the total current in this band?

   w. Use just the electron current (part b), as (x) is wrong.
   x. Use just the hole current (part d), as (w) is wrong.
   y. Add the electron and hole currents to get the total current.
   z. Use either the electron or the hole current; they necessarily give the same answer.

5. A&M 13.2

6. A&M 14.1

7. A&M 14.2 (one-line answer). The horizontal axis is \(1/H\).

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\(^1\) The course textbooks all comment that scattering makes this oscillation unobservable, which may be true, but it reminds me of the A.C. Josephson effect in a superconductor–normal-metal–superconductor junction, in which again a D.C. voltage drives an alternating current. As far as I can tell, however, the mechanisms have nothing to do with each other.